IMPULSIVE SYMMETRICAL CAVITATIONAL FLOW PAST A GRID OF PLATES

(UDAR RESHETKI PLASTIN PRI SIMMETRICHNOM KAVITATSIONNOM OBTEKANII)

PMM Vol.22, No.4, 1958, pp.565-568

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(Received 27 January 1957)

The steady symmetric cavitational flow past a plate in a stream of ideal liquid has been investigated by Gurevich [2], following Efros [1]. Apparently, this flow can be considered as a cavitational flow about a grid of plates of fineness l/L.

1. Boundary value problem. We consider a plate, set normally to a current of ideal liquid, which is bounded by parallel walls (Fig. 1). We assume that the plate suddenly attains a forward velocity v_1 into the stream (frontal impact). The resulting impulsive flow has a velocity potential ϕ, connected with the impulsive pressure p and liquid density ρ by the relation

$$p = -\rho\varphi \tag{1.1}$$

The complex potential of the impulsive flow is $w = \phi + i\psi$. The harmonic functions $\phi(x, y)$, $\psi(x, y)$, defined in the plane of flow z, must satisfy the following boundary conditions:

1. On a free surface, $p = 0$ and therefore $\phi = 0$.

2. The normal velocity on the plate $\delta\phi/\partial n = v_1$ is known.

3. The x axis and walls are streamlines $\psi = \text{const}$.

Moreover, physical considerations require that the complex-conjugate velocity dw/dz approaches a definite value at infinitely distant points in the stream, tends to infinity at the edges of the plate and approaches zero on the streamline D.

The flow region between the wall and the x axis is mapped by conformal transformation into the upper right quadrant (Fig. 2) of the plane of the transformed variable $u = \xi + i\eta$. Corresponding points in Fig. 1 and 2 are denoted by the same letters. The transformation is defined, following

Ref. [2], by

$$\frac{dz}{du}=\frac{N(u+e)^2}{v_0u(u^2-a^2)(u^2-h^2)}\sqrt{\frac{1+u}{1-u}} \tag{1.2}$$

where N is a physical constant, v_0 the velocity magnitude corresponding to steady cavitational flow with a free surface, and e is a parameter, which may be expressed in terms of the basic parameters a, h by the relation

$$e=\frac{ah(\sqrt{\alpha}-\sqrt{\chi})}{h\sqrt{\chi}-a\sqrt{\alpha}}\qquad \alpha=\sqrt{\frac{1-a}{1+a}},\qquad \chi=\sqrt{\frac{1-h}{1+h}} \tag{1.3}$$

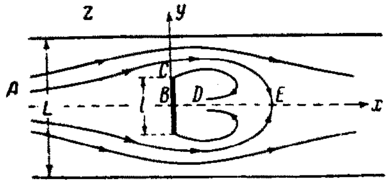

Fig. 1.

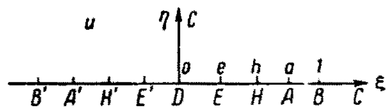

Fig. 2.

The boundary conditions satisfied by the function dw/du in the u plane are

$$\operatorname{Im}\frac{dw}{du}=\begin{cases}0 & \text{when } \xi=0,\ 0<\eta<\infty\\ v_1\,|dz/du| & \text{when } \eta=0,\ 1<\xi<\infty\\ 0 & \text{when } \eta=0,\ 0<\xi<1\end{cases} \tag{1.4}$$

The first of conditions (1.4) enables us to continue dw/du by symmetry into the second quadrant of the upper half u-plane. The analytic function dw/du in the upper half-plane then satisfies the boundary conditions

$$\operatorname{Im}\frac{dw}{du}=\begin{cases}v_1\,|dz/du| & \text{when } \eta=0,\quad 1<\xi<\infty \qquad (1.5)\\ -v_1\,|dz/du| & \text{when } \eta=0,\ -\infty<\xi<-1 \qquad (1.6)\\ 0 & \text{when } \eta=0,\ -1<\xi<1 \qquad (1.7)\end{cases}$$

and is determined by Schwartz's integral,

$$\frac{dw}{du}=\frac{1}{\pi}\int_{-\infty}^{\infty}\operatorname{Im}\frac{dw}{du}\,\frac{d\xi}{\xi-u}=\frac{2v_1|N|}{\pi v_0}\int_1^{\infty}\frac{(e+\xi)^2\sqrt{(\xi+1)/(\xi-1)}}{(\xi^2-a^2)(\xi^2-h^2)(\xi^2-u^2)}$$

Integrating, we obtain

$$\frac{dw}{du}=\frac{2v_1|N|}{\pi v_0(u^2-a^2)(u^2-h^2)}\Big[[(u^2-a^2)K_1+(u^2-h^2)K_2-$$

$$-\frac{e^2+u^2(1+2e)}{u\sqrt{u^2-1}}\ln\frac{\sqrt{(u+1)/(u-1)}+1}{\sqrt{(u+1)/(u-1)}-1}+i\,\frac{\pi(e+u)^2}{2u}\sqrt{\frac{u+1}{u-1}}\ \Big]$$

$$K_1=\frac{1}{\chi h(a^2-h^2)}\left[\frac{\pi}{2}(e+h)^2-2\,\frac{e^2+h^2(1+2e)}{1+h}\,\mathrm{arc\,tg}\chi\right] \tag{1.8}$$
$$K_2=\frac{1}{\alpha a(h^2-a^2)}\left[\frac{\pi}{2}(e+a)^2-2\,\frac{e^2+a^2(1+2e)}{1+a}\,\mathrm{arc\,tg}\,\alpha\right]$$

This expression for dw/du satisfies all boundary conditions, and the physical conditions to be fulfilled by dw/dz.

2. Determination of the impulsive force. Let J_x be the total impulsive force exerted by the liquid on the plate. Then from Fig. 1 and equation (1.1) we have

$$J_x=-2i\int_{BC}p\,dz=2i\rho\int_{BC}\varphi\,dz$$

Integrating by parts in the transformed plane, and noting that $\phi=0$ at the ends of a plate, we obtain

$$J_x=-2i\rho\int_1^\infty z\,\frac{d\varphi}{du}\,du=-2i\rho\int_1^\infty z(u)\,\mathrm{Re}\,\frac{dw}{du}\,du \tag{2.1}$$

where $\mathrm{Re}(dw/du)$ is to be determined by (1.8). The value of $z(u)$ is to be determined.

Since the points B in the z and u planes correspond, integration of (1.2) gives

$$z(u)=-i\,\frac{N}{v_0}\left[A\,\mathrm{arc\,tg}\,t+\frac{B_+}{\chi}\,\mathrm{arc\,tg}\,\chi t+B_-\chi\,\mathrm{arc\,tg}\,\frac{t}{\chi}+\right.$$
$$\left.+\frac{C_+}{\alpha}\,\mathrm{arc\,tg}\,\alpha t+C_-\alpha\,\mathrm{arc\,tg}\,\frac{t}{\alpha}-\frac{\pi}{2}(A+B_-\chi+C_-\alpha)\right] \tag{2.2}$$
$$t=\sqrt{\frac{u+1}{u-1}},\quad A=\frac{2e^2}{a^2h^2},\quad B_\pm=-\frac{1}{a^2-h^2}\left(1\pm\frac{e}{h}\right)^2,\quad C_\pm=\frac{1}{a^2-h^2}\left(1\pm\frac{e}{a}\right)^2$$

Since the points C in the z and u planes correspond

$$N=\frac{1}{2}\left[\frac{\pi}{4}A+\left(B_-\chi-\frac{B_+}{\chi}\right)\mathrm{arc\,tg}\,\chi+\left(C_-\alpha-\frac{C_+}{\alpha}\right)\mathrm{arc\,tg}\,\alpha\right]^{-1}v_0l=N_0v_0l \tag{2.3}$$

where $N_0=N/v_0l$ is a dimensionless coefficient. Finally, as in Ref.[2], the fineness of the grid l/L and the cavitation number λ are determined from the equations

$$\frac{l}{L} = \frac{\alpha a^2 (a^2 - h^2)}{\pi (a + e)^2 N_0}, \qquad \lambda = \frac{2a (a^2 + e^2 + 2e)}{(1 - a)(a - e)^2} \tag{2.4}$$

If the variable $u = \sec\theta$ is substituted into (2.1), (1.8) and (2.2), the impulsive force J_x is given in a form convenient for numerical integration. The dimensionless coefficient $\mu^0 = J_x/\rho v_1 l^2$ for one plate of the grid was calculated for different values of the parameters a, h. Further, graphs for l/L and λ were drawn with use of formulas (2.4) and (2.3).

Fig. 3 shows the relation between μ^0 and λ for different values of l/L. Curve 1 corresponds to a plate grid with an infinite cavitational zone, curve 2 to cavitational flow past a plate in an infinite stream. It is evident that μ^0 increases with increase in λ when l/L is constant. The influence of l/L on μ^0 becomes more significant as l/L increases. Two important special cases are discussed here.

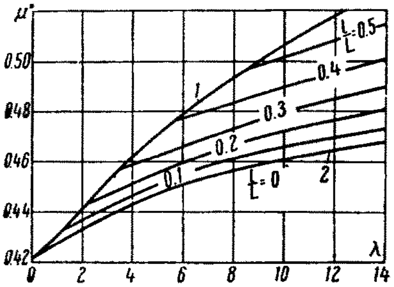

Fig. 3.

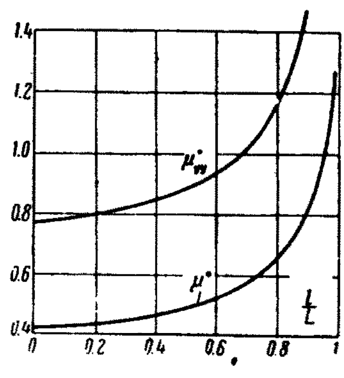

Fig. 4.

3. Impulsive flow past a grid of plates with separation. When $h \to 0$ and $e \to 0$ the base of the streamline D disappears to infinity, and we have flow past a plate grid with a separated stream. After introducing these limits into (1.3), (1.8), (2.2), (2.3) and (2.4), the values of μ^0 were computed for different values of the independent parameter a ($0 < a < 1$). The graphs of l/L (Fig. 4) and λ were then drawn (curve 1 in Fig. 3). A graph of the dimensionless coefficient μ_{yy}^0 for the plate grid in continuous flow is shown in Fig. 4 for comparison; it was computed from a formula in Ref. [3]:

$$\mu_{yy}^\circ = \frac{\mu_{yy}}{\rho l^2} = -\frac{2}{\pi}\left(\frac{L}{l}\right)^2 \ln\cos\frac{\pi}{2}\frac{l}{L}$$

Fig. 4 shows that large increases in μ^0 and μ_{yy}^0 occur only when $l/L > 0.7$. It is interesting to note that $\mu_{yy}^0/\mu^0 \approx 2$ for any value of l/L. This may be explained by the fact that the disturbance of the liquid is caused mostly by the front part of a plate when flow about it separates, while both parts of the plate are involved during continuous flow.

When the grid mesh is wide ($a \to 0$),

$$\frac{l}{L} \approx \frac{\pi + 4}{2\pi} a^2$$

If, for small values of a, we expand $z(u)$ in powers of a up to terms of order a^4 and expand $\mathrm{Re}(dw/du)$ up to terms of order a^3 we obtain the following formula for μ^0, which is accurate up to (a^2):

$$\mu^{\circ} \approx \frac{1}{\pi(\pi+4)^2} \int_0^{1/2\pi} [A_1A_2 + a^2(4A_1B_2 + \frac{1}{2}A_1A_2 + 4A_2B_1)]\, d\theta = 0.4224 + 0.8697a^2$$

$$A_1 = 2\theta + 4\sin\theta + \sin 2\theta, \qquad A_2 = (2\pi + 4)\sin\theta + 4\cos^2\theta \ln \operatorname{ctg} \frac{\pi - 2\theta}{4}$$

$$B_2 = \frac{3\pi + 8}{12}\sin\theta + \left(\frac{\pi}{2} + 1\right)\sin\theta\cos^2\theta + \cos^4\theta \ln \operatorname{ctg}\frac{\pi - 2\theta}{4}$$

$$B_1 = \frac{3}{8}\theta + \frac{5 + \cos 2\theta}{6}\sin\theta + \frac{4 + \cos 2\theta}{16}\sin 2\theta \qquad (3.1)$$

This formula is valid for $0 < l/L < 0.001$. When $a \to 0$ streamlined flow past a plate in an unlimited stream is obtained, for which (3.1) gives the value $\mu^0 = 0.4224$, previously found in Ref. [4].

4. Impulsive cavitational flow past a plate in an unlimited stream. When $h \to a$ the walls bounding the cavitational flow recede to infinity. In this case, from (1.3) and the inequality $0 < e < 1$ we find that the limits of variation of the independent parameter a are $0 < a < 1/2(\sqrt{5} - 1)$. In the limiting case when $h \to a$ in the basic formulas, values of μ^0 were computed for different values of a. The graph of λ was then drawn (curve 2 in Fig. 3). It is evident that the value of μ^0 increases very little when $\lambda > 8$. Note that for a very short cavitation zone $\lambda \to \infty$ and μ^0 must approach $1/4\pi$, the value of the coefficient determined in Ref. [3] for continuous flow past a plate. For fully developed cavitational flow ($a \to 0$, $\lambda \to 0$), proceeding as in Section e, we obtain the approximate formula

$$\mu^{\circ} \approx 0.4224 - 0.0378\, a^2$$

which is valid when $0 < \lambda < 0.1$. When $a \to 0$ the value $\mu^0 = 0.4224$ given in Ref. [4] is obtained for flow past a plate with separation.

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Translated by S.K.